APPENDIX A: Evaporation and Sheathing

*In-vitro*

To ascertain carefully the role of evaporation in Schirmer-strip wetting dynamics, *in-vitro* and *in-vivo* imbibition experiments were performed at 40% relative humidity and ambient temperature. For the *in-vitro* experiments, Whatman #41 filter-paper sheets (Capitol Scientific, Austin, TX) were cut into 5-mm wide, 65-mm long strips, and marked in light pencil 5 mm from one end. Strips were affixed vertically with the penciled end facing downward. A 325-mL Pyrex Petri dish filled with distilled water containing dilute sodium fluorescein was slowly raised to touch the strip. Dyed wetting lengths were then captured in time on a Smart phone (Galaxy S7, Samsung, Seoul, South Korea). Imbibition lengths were recorded over roughly a 10-11 min time span to at least the 50-mm position. Visual checks on water imbibition with and without sodium fluorescein dye confirmed that water and dye fronts coincide.

Figure A1 reports the average of 8 *in-vitro* trials of unsheathed strips in terms of wetting length versus square root of time, as suggested by a Washburn analysis of wetting dynamics in porous media from an unlimited source\(^1\)

\[
\frac{dL_p(t)}{dt} = \frac{Q_s(t)}{\phi \omega \delta} = \frac{\kappa}{\phi \mu L_p} \left( \frac{2 \gamma}{R_p} \right) - \frac{\kappa}{\phi \mu} \hat{\rho} g
\]  

(A1)

where \(\kappa\) is the strip hydraulic permeability, \(\hat{\rho}\) is the mass density of tear, \(\mu\) is the viscosity of tear, and \(g\) is the acceleration of gravity. With \(L_p(0) = 0\), the solution for short times is

\[
L_p(t) = \left( \frac{4 \kappa \gamma}{\phi \mu R_p} \right)^{1/2} \sqrt{t}.
\]  

(A2)
Figure A1. Wetting dynamics of unsheathed Whatman #41 paper strips at ambient temperature and 40 %
relative humidity. Error bars indicate standard deviation for 8 repeat trials.

The Washburn result in Eqn. A2 explains the choice of the square-root time abscissa in Figure A1. The dashed line in Figure A1 corresponds to numerical solution of Eqn. A1 (ODE45 in Matlab, Mathworks, Inc. Natick, MA) using the physical parameters listed in Table 1. Clearly, there is significant underestimation of the wetting-kinetic data from Eqn. A1, presumably due to evaporation.

To assess whether the observed deviations from Washburn kinetics are indeed due to evaporation, we rewrite the mass balance in Eqn. A1 to include evaporative loss:

\[
\frac{dL_p(t)}{dt} = \frac{Q_g(t)}{\phi w \delta} = \frac{\kappa}{\phi \mu L_p} \frac{2\gamma}{R_p} - \frac{\kappa}{\phi \mu} \hat{g} - \frac{2\hat{J}_\mu L_p}{\delta \hat{\rho}}
\]  
(A3)
where \( \dot{J}_E \) is the mass evaporation flux (mass evaporated/area/time). The minus sign appearing before the evaporation term on the right side of Eqn. A3 indicates slowing of the wetting front. \( \dot{J}_E \) is not a constant but depends on the wetted length of the strip among other variables including temperature, relative humidity, and airflow.\(^{2,3}\) To quantify \( \dot{J}_E \), we adopt the analysis in Appendix A of Telles et al.\(^2\) That analysis is invoked with the one change that the characteristic length appearing in the Nusselt number is modified to: 

\[
L_c = \frac{\varphi w L_F}{2(L_F + w)}.
\]

Eqn. A3 is then solved numerically by a Runge-Kutta algorithm (ODE45 in Matlab, Mathworks, Inc. Natick, MA). All physical parameters are given in Table 1 or in Appendix A of Telles et al.\(^2\) Thus, the solid curve in Figure A1 is predicted with no adjustable constants. Agreement between theory and experiment is excellent demonstrating that wicking-imbibition kinetics is obeyed.

Since Whatman #41 paper strips are physically interchangeable with the supplied Schirmer strips, the results Figure A1 indicate that evaporation is also important \textit{in vivo} because large, noticeable deviations of the experimental data from Eqn. A1 begin already at roughly 100 s in Figure A1, well within the operating window of the \textit{in-vivo} trials. Tear evaporation from Schirmer strips becomes more evident at relative humidities lower than 40 %, slowing imbibition kinetics even more than that in Figure A1.

To reduce evaporation from Schirmer strips and obtain quantitative basal tear production, we follow others in sheathing the strips.\(^2,4\) We conducted identical \textit{in-vitro} experiments to those in Figure A1 after sheathing the Whatman #41 filter paper strips with Wexford Packing Duct Tape, as described in the main text. However, to mimic the dyed Schirmer strips and to accentuate wetted-front visualization, FD&C Blue 1 dye was added to the aqueous solution instead of
sodium fluorescein. Sheathed strips were tested immediately after sheathing to prevent contamination by tape adhesive. Imbibition continued until a wetted length of 50 mm was reached. Experiments spanned roughly 4-5 min.

Figure A2 displays as open circles the resulting experimental *in-vitro* wetting dynamics for the sheathed strips. Again, standard-deviation error bars are shown here for 5 separate trials. The dashed solid line corresponds to Eqn. A1 (i.e., for zero evaporation) with no adjustable constants. Excellent agreement is obtained for the first few min confirming success in protection against evaporation. Systematic errors appear, however, after about 3 min with wetting-dynamics data exceeding that predicted.

**Figure A2.** Wetting dynamics of sheathed Whatman #41 paper strips at ambient temperature and 40% relative humidity. Error bars give standard deviation for 5 repeat trials.
Careful examination of the wetting front when the systematic errors commence shows a concave wetting-front shape. Before the concave shape evolves and for unsheathed strips, the wetting front is flat, as expected in a porous medium. We postulate that sealing the strip by transparent tape creates an unavoidable narrow channel at the strip edges between the two layers of tape just before they seal. This open channel provides an imbibition path that is less flow resistant than that in the pores of the porous-paper strip. Water imbibes faster into the two parallel side channels than into the strip and eventually increases the speed of the wetting front. No such wetting-front enhancement was observed in the on-eye experiments because wetting lengths are shorter. We conclude that sheathing Schirmer strips with transparent tape effectively eliminates tear evaporation.

![Figure A3](image.png)

**Figure A3.** Unsheathed and sheathed strip comparison on same subject, same trial. Unsheathed strip shows significantly shorter 5-min wetted length, which indicates sheathing is necessary for accurate data. In both cases, dye-front feathering is observed behind the water front.

**In-vivo**

Evaporation also infects *in-vivo* Schirmer-strip wetting kinetics. Figure A3 captures on-eye wetting dynamics of a single person for two Schirmer strips: left unsheathed and right sheathed. Note that the wetting front on the sheathed (right) strip reaches 32 mm in 5 min, whereas the
unsheathed (left) strip attains only 19 mm during the same 5-min period. We assert that the faster imbibition rate in the sheathed strip confirms significant reduction in evaporative tear loss due to the sheathing process. We, thus, confirm the importance of evaporation in Schirmer-tear tests, and we follow the recommendation of Telles et al.\textsuperscript{2} to apply sheathing.

REFERENCES


