Appendix E-1

The Trigonometric Algorithm Defining the True 3D Acetabular Cup Orientation

Correlation Between Measured and Calculated Cup Orientation Angles

Three-dimensional (3D) geometrical acetabular cup orientation can be defined in the anatomical planes by coronal inclination, transverse version, and sagittal tilt of the cup (see Figs. 1 and 2 in article). In modern total hip arthroplasty, most cups are almost a perfect hemisphere and have a circular metal wire inserted at the outer circumference of the cup. This wire or the outer rim of the cup is visible on computed tomography (CT) or radiographic imaging of the cup.

Because the shape of the cup is hemispherical, with a constant diameter, and these angles are measured in 3 perpendicular planes, these are trigonometrically related to each other. This supplement describes the trigonometric formulas of these angles, which defines the orientation of the cup in a unique manner.

Definitions and Abbreviations

Three-dimensional coordinate system:
Mediolateral (transverse) axis (X)
Craniocaudal (longitudinal) axis (Y)
Anteroposterior (sagittal) axis (Z)
Opposite (O)
Adjacent (A)
Diameter of the cup (S)

Coronal Inclination (Fig. E-1A): Coronal inclination (α) is defined as the spatial angle between the transverse axis (X) and the cup in the coronal plane at the center of the femoral head. The coronal inclination has a complementary angle (α'), which is the angle between the longitudinal axis (Y) and the cup. The sum of those 2 angles is 90°.

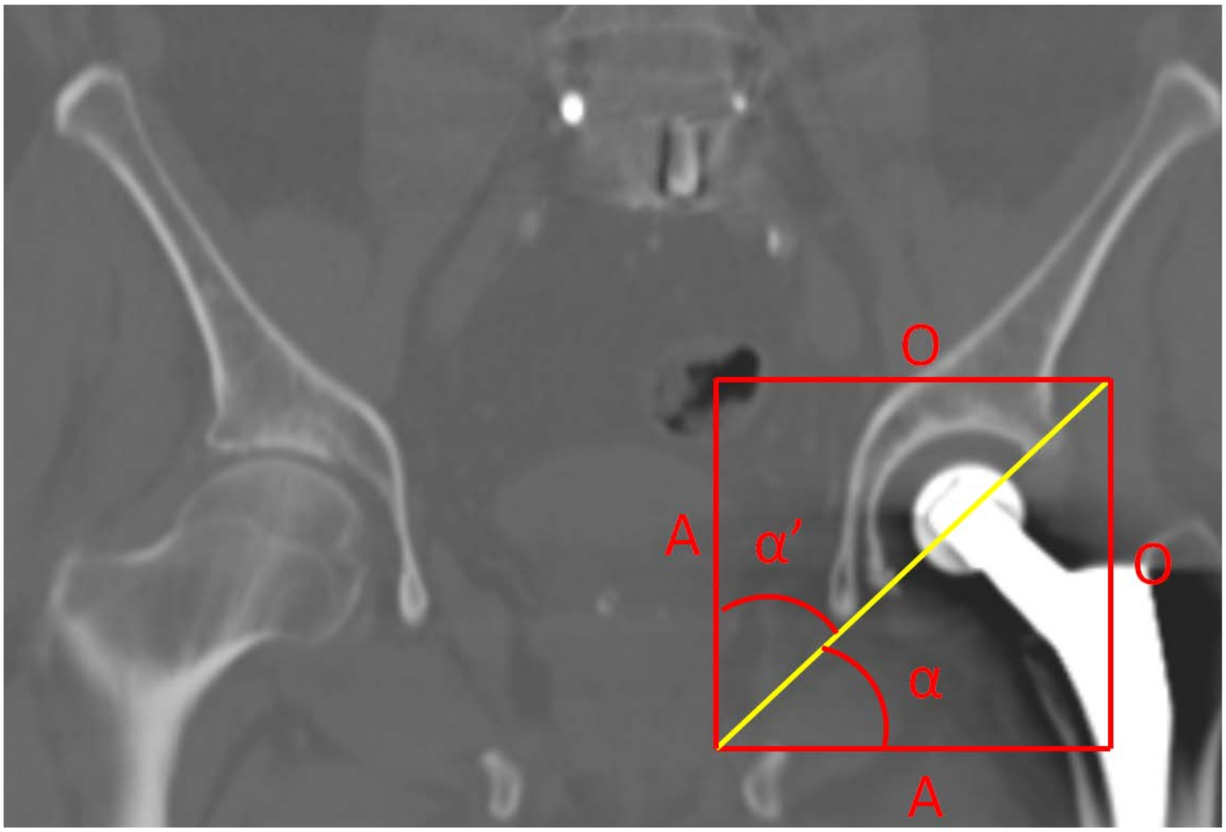

Fig. E-1-A

Sagittal Tilt (Fig. E-1B): Sagittal tilt (γ) is defined as the spatial angle between the sagittal axis (Z) and the metal wire projected onto the sagittal plane at the center of the femoral head. The sagittal tilt has a complementary angle (γ'), which is the angle between the longitudinal axis (Y) and the metal wire. The sum of those 2 angles is 90°.

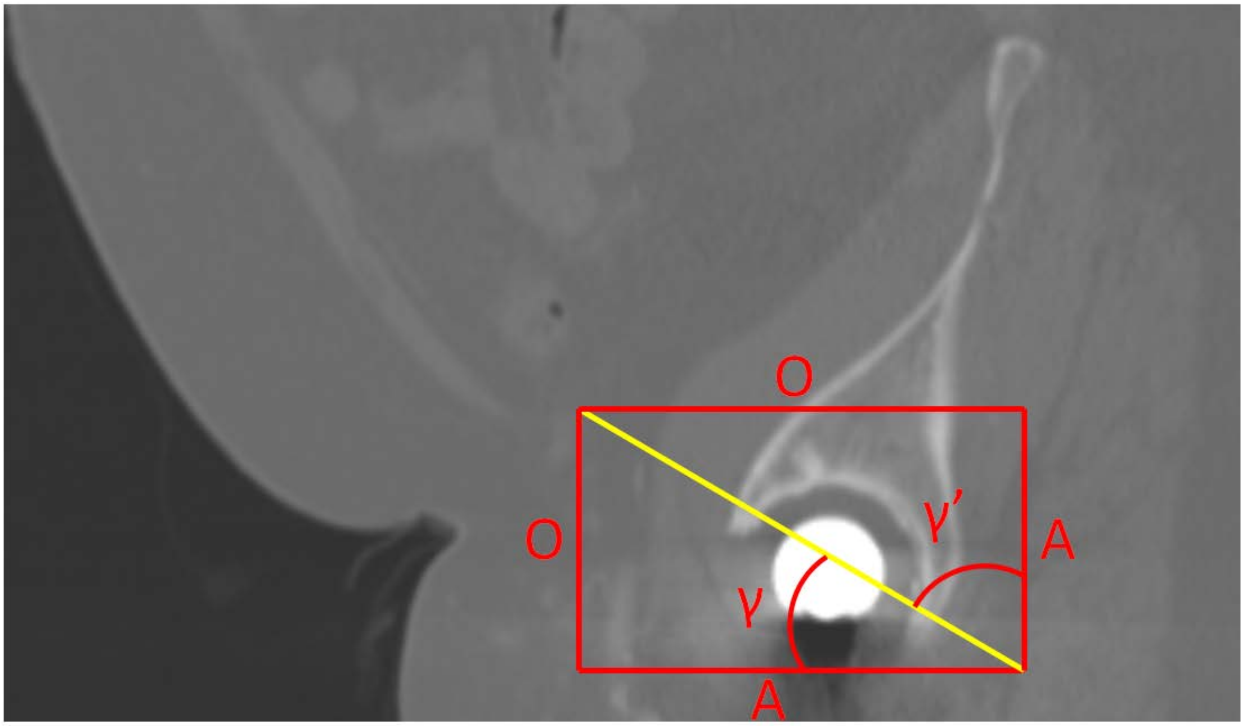

Fig. E-1-B

Transverse Version (Fig. E-1C): Transverse version (β) is defined as the spatial angle between the sagittal axis (Z) and the metal wire projected onto the transversal plane at the center of the femoral head. The transverse version has a complementary angle (β'), which is the angle between the transverse axis (X) and the metal wire. The sum of those 2 angles is 90°.

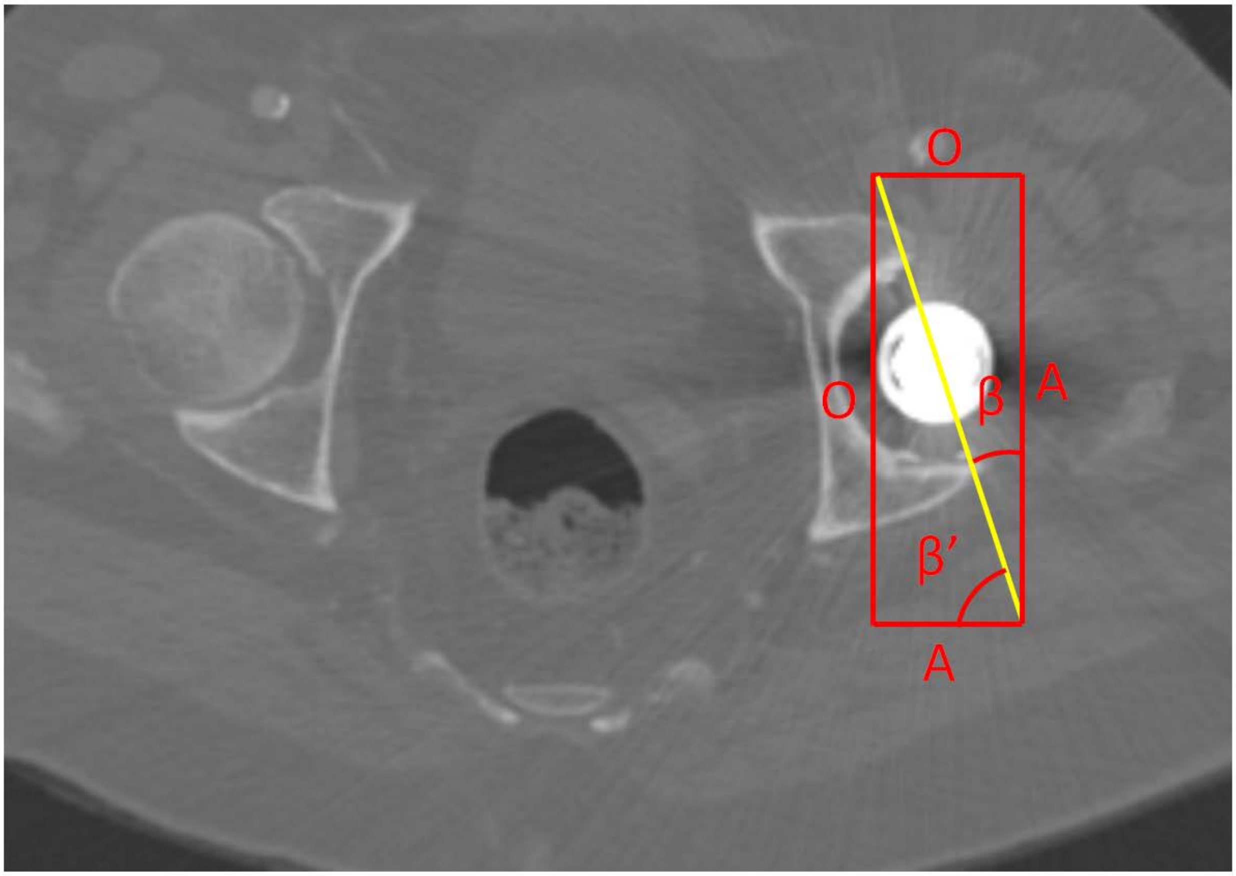

Fig. E-1-C

Mathematical Rationale of the Trigonometric Algorithm

The orientation of the cup can be described as a vector:

$$\overrightarrow{\text{Acetabular cup orientation}} \quad \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

The vectors of the cup orientation projected onto the coronal, sagittal, and transversal planes are, respectively:

$$\overrightarrow{\text{Inclination}} \begin{bmatrix} X \\ Y \\ 0 \end{bmatrix} \qquad \overrightarrow{\text{Version}} \begin{bmatrix} X \\ 0 \\ Z \end{bmatrix} \qquad \overrightarrow{\text{Tilt}} \begin{bmatrix} 0 \\ Y \\ Z \end{bmatrix}$$

By using the trigonometric equations, those angles could be described as follows, including their complementary angles:

Inclination (α):

$$\tan\alpha = \frac{O}{A} = \frac{Y}{X}$$
$$\sin\alpha = \frac{O}{S} = \frac{Y}{S}$$
$$\cos\alpha = \frac{A}{S} = \frac{X}{S}$$

Version (β):

$$\tan\beta = \frac{O}{A} = \frac{Y}{X}$$
$$\sin\beta = \frac{O}{S} = \frac{Y}{S}$$
$$\cos\beta = \frac{A}{S} = \frac{X}{S}$$

Tilt (γ):

$$\tan\gamma = \frac{O}{A} = \frac{Y}{X}$$
$$\sin\gamma = \frac{O}{S} = \frac{Y}{S}$$
$$\cos\gamma = \frac{A}{S} = \frac{X}{S}$$

Complementary inclination (α'):

$$\tan\alpha' = \frac{O}{A} = \frac{Y}{X}$$
$$\sin\alpha' = \frac{O}{S} = \frac{Y}{S}$$
$$\cos\alpha' = \frac{A}{S} = \frac{X}{S}$$

Complementary version (β'):

$$\tan\beta' = \frac{O}{A} = \frac{Y}{X}$$
$$\sin\beta' = \frac{O}{S} = \frac{Y}{S}$$
$$\cos\beta' = \frac{A}{S} = \frac{X}{S}$$

Complementary tilt (γ'):

$$\tan\gamma' = \frac{O}{A} = \frac{Y}{X}$$
$$\sin\gamma' = \frac{O}{S} = \frac{Y}{S}$$
$$\cos\gamma' = \frac{A}{S} = \frac{X}{S}$$

The cup is a circular hemisphere with a constant diameter. Therefore, mathematically seen, one is able to change the diameter by 1. This gives the opportunity to give an equation by which, if 2 angles are known, the third one could be calculated. These equations describe the relationship between the 3 angles (α, β, and γ; see Table I and Fig. 5 in article):

$$Inclination = \arctan(\frac{\tan Version}{\tan Tilt})$$
$$Version = \arctan(\tan Tilt \times \tan Inclination)$$
$$Tilt = \arctan(\frac{\tan Version}{\tan Inclination})$$

Practical Considerations and Potential Pitfalls

- Mathematically, this trigonometric algorithm can be used for a symmetrical, hemispherical cup.
- Measurement should be performed by consistently using the circular metal wire around either the cup or the acetabular rim.
- The anatomical planes should be used because these are exactly perpendicular to each other. Multiplanar reconstructions acquired using 3D imaging modalities or images acquired with use of biplanar radiography seem most suitable.
- We recommend using the algorithm with caution in cases in which the sagittal tilt and transverse version are approaching 0°. A small measuring error affects a small positive or negative tilt and version to a great extent. This could have an influence on the proportions between them, affecting the outcome of the algorithm.
- For negative values of transverse version (retroversion), sagittal tilt is negative as well (retrotilt), and vice versa. Furthermore, if negative sagittal tilt and negative transverse version are present, the following equations are required:

$$Inclination = \arctan(\frac{\tan(90 - Version)}{\tan(90 - Tilt)})$$
$$Version = 90 - (\arctan(\tan(90 - Tilt) \times \tan Inclination))$$
$$Tilt = 90 - (\arctan(\frac{\tan(90 - Version)}{\tan Inclination}))$$

TABLE E-1 Transverse Version for Given Coronal Inclinations and Sagittal Tilt*

Calculated transverse version	Inclination								
Sagittal tilt	0°	1°	15°	30°	45°	60°	75°	89°	90°
0°	0	0	0	0	0	0	0	0	—
15°	0	0.3	4.1	8.8	15	24.9	45	86.3	—
30°	0	0.6	8.8	18.4	30	45	65.1	88.3	—
45°	0	1	15	30	45	60	75	89	—
60°	0	1.7	24.9	45	60	71.6	81.2	89.4	—
75°	0	3.7	45	65.1	75	81.2	85.9	89.7	—
90°	—	—	—	—	—	—	—	—	—

*All values are given in degrees.