APPENDIX

To analyze the contrast reduction in the MxP, we assume that users see the multiplexed scene of a target image, a Gabor patch with maximum \( M \) and minimum \( m \) luminance, over a uniform background luminance of \( (M + m)/2 \), in the shifted view. The see-through view is the view of a blank area through the flat elements with uniform luminance \( B \) (which may be different from the background of the Gabor patch). The Michelson contrast of the target \( C \) observed without the multiplexing prism is \( (M - m)/(M + m) \).

The luminance of the target in the shifted view is reduced by the factor of the aperture ratio, \( r \). Similarly, the luminance of the blank image seen through the flat elements \( (B_F) \) is reduced by \( (1 - r) \). As a result, the contrast of the target in the multiplexed scene \( C_M \) is

\[
C_M = \frac{M_p + B_F - (m_p + B_F)}{M_p + B_F + (m_p + B_F)} = \frac{M_p - m_p}{M_p + m_p + 2B_F} = \frac{r(M - m)}{r(M + m) + 2B_F}.
\]

(1)

where \( M_p = rM, m_p = rm \), and \( B_F = (1 - r)B \). The contrast of the target seen through the multiplexing prism is reduced by \( C_R \), the contrast reduction factor,

\[
C_R = \frac{C_M}{C} = \frac{r(M - m)/r(M + m) + 2B_F}{(M - m)/(M + m)} = \frac{r(M + m)}{r(M + m) + 2B_F}.
\]

(2)

The contrast reduction factor \( C_R \) is highly affected by the luminance of the see-through view \( (B_F) \) that in turn is controlled by the uniform background \( (B) \) and the aperture ratio \( (r) \). If the see-through view is dark \( (B << m) \), there is little contrast reduction \( (C_R \approx 1) \). Similarly, if the aperture ratio is such that the width of the prism elements is much wider than the width of the flat elements \( (r \approx 1) \), as is the case with a conventional prism, the contrast is not reduced. However, a more likely situation is where the luminance of the see-through view is about the same as the
luminance of the background seen in the shifted view. In this situation,

\[ B \approx (M + m)/2 \quad \text{and} \quad B_F = (1 - r)(M + m)/2. \]

Therefore, the contrast reduction factor \((C_R)\) is approximately the same as the aperture ratio \((r)\) of the multiplexing prism.

In addition to the flux attenuation effect (caused by the aperture ratio, \(r\)), the luminance through the prism or flat elements is also affected and reduced by the optical transmittance of each element \((t_P \text{ for the prism elements and } t_F \text{ for the flat elements}).\) The optical transmittance of the prism elements is not a constant but a function of the angle of incidence, as shown in Fig. 4B. When the luminance of the background in the see-through view is the same as the shifted views, the contrast reduction factor \((C_R)\) will be modified by the optical transmittance:

\[
C_R = \frac{t_p r (M + m)}{t_p r (M + m) + 2t_F B_F} = \frac{t_p r (M + m)}{t_p r (M + m) + t_F (1 - r)(M + m)} = \frac{t_p r}{t_p r + t_F (1 - r)}.\]  

(3)

The flat elements always have fixed (and high) transmittance \((t_F)\) regardless of the angle of incidence (Fig. 4B), whereas the optical transmittance of the prism elements \((t_P)\) varies with the angle of incidence\(^{14}\) and is mostly lower than the optical transmittance of the flat elements \((t_F)\). Therefore, the contrast reduction factor of the target in the shifted view may be lower than the aperture ratio \((r)\), further reducing the target contrast. If the transmittance in the prism elements is close to that of the flat elements \((t_P \approx t_F)\), as in the eyeward prism serrations configuration (Fig. 4B), the contrast reduction is approximately the same as the aperture ratio \((C_R \approx r)\) as shown in Fig. A1.
Figure A1. Contrast reduction of the target in the shifted view of a 57Δ multiplexing prism (MxP) with various aperture ratios (r = 40%, 50%, and 60%). Due to the variation of the transmittance in the prism elements and fixed transmittance in the flat elements, the contrast reduction in the MxP is controlled by the transmittance variations in the prism elements. In the eyeward prism serrations (EPS) configuration, the contrast is almost the same as the aperture ratio, though it drops close to zero near TIR in higher eccentricities. In the OPS configuration, however, the wide variation in transmittance with gaze angle results in highly reduced contrast, especially when approaching the critical angle of incidence.

To compensate for the contrast reduction caused by the transmittance difference, the aperture ratio should be higher than the desired contrast reduction factor. At normal incidence, for example, the transmittance of a flat element of 57Δ PMMA multiplexing prism is 92 % but a 57Δ outward prism serrations prism element has only 82% transmittance, which results in 47% of the original contrast with multiplexing prism aperture ratio of 50%. An aperture ratio of 53% multiplexing prism would result in the intended 50% contrast reduction. This difference is too small to be meaningful. Although an angle of incidence close to the critical angle of incidence results in highly reduced transmittance and contrast, for most of the angular range, the aperture ratio is the main factor in the contrast reduction in the outward prism serrations configuration.
If the target/object-of-interest is in the see-through view and the shifted view has a blank image of the same background luminance, the contrast reduction factor of the see-through view is $1 - C_R$. Due to the optical transmittance reduction in the prism elements (blank background), the contrast of the target in the see-through view is little higher than the aperture ratio.